Time-variant failure probability of critical slopes under strong rainfall hazard including mitigation effects

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The time-variant failure probability for critical slopes under strong rainfalls, which might cause reductions on the soil shear strength, is calculated and compared to the target failure probability. The soil properties and rainfall characteristics are considered as random and the correlation between rainfall intensity and duration is included to assess the impact of water infiltration on the slope failure probability. The failure probability, defined as the probability that the safety factor is less than 1, is calculated through a Monte Carlo simulation process. The paper emphasizes the importance of the time during the water infiltration into the soil, as the rainfall sequence occurs. The target failure probability, derived from the minimum expected life-cycle cost, is compared to the slope failure probability to decide if the slope requires mitigation measures. If mitigation is required, the slope model is modified and a new annual failure probability is calculated. The slope annual failure probability, for 3 sites under strong rainfall hazard, is found to be around 0.78. However, by introducing anchor bars, the failure probability reduces to 0.037. The proposed formulation allows for the risk-based prioritization for the attention of a set of slopes at several sites and the consideration of mitigation actions.

Keywords: slope stability; environment, risk and probability analysis, safety factors, reinforcement

1 Introduction

Landslides occur all around the world as reported by Larsen and Simon (1993), Glade (2001), Marengo and Granados (2011) and, sometimes, the failure consequences involve fatalities and large economic losses: Lin and Jeng (2000), Alcántara (2004, 2008). The soil stability analysis, as started by Fellenious (1927) and Terzaghi (1943), considered certain assumptions regarding mainly the failure path. The classical 2D slice method with circular critical slip surface has been modified since then. Some researchers like Fukuoka (1980), Orense (2004) and Zhang et al., (2005) have studied the change on soil properties after the occurrence of a strong rainfall.

A general equilibrium equation has been described by Fredlund (2007) through a hazard management system to cope with the failure consequences. The water infiltration through the soil have been considered through the Richard´s equation for unsaturated flow, Richards (1931), to obtain the effective pore pressure, among other variables.

The fact that the soil partial saturation is more unfavourable than the full saturation has been recognized by researchers like Fredlund et al. (2012). The geo-hydro-meteorological hazard has been studied from many points of view: one of them is the challenging issue of the amount of water that infiltrates the soil and produces a sudden increment on the slope vulnerability, as shown by White and Singham (2012).

The relationship between intensity and duration of rainfalls that provoke landslides has been explored in the work by Guzzeti et al. (2008). Several approaches have been proposed to deal with the time consuming shortcomings of classical and crude Monte Carlo simulation techniques. Modern numerical techniques, like genetic algorithms, chaos expansion polynomials and variance reduction procedures, among others, are being promoted by several research groups like Luo et al. (2012) and Jiang and Huang (2016).

The potential of Bayesian networks and copulas are currently being used to consider to model correlations among variables and to update the probabilities, especially when the resistance of the soil structure suddenly changes to increase the soil vulnerability, as studied by Wu (2013).

A safety factor, defined in terms of work rates, has been used to calculate the *pdf* of the slope safety factor and a normal distribution was found to approximate well the internal friction angle for values up to 16° as described by Florkiewicz and Kubzdela (2013). Ground improvement achieved by vertical drains and surcharge preloding has been assessed by using stochastic cost optimization (Kim et al., 2014).

In Mexico the lanslide at La Pintada, Guerrero caused 78 deaths and motivated to do in-deep studies for mitigation of this hazard as reported by Alcántara-Ayala, et al., 2017. A new procedure based on recursive algorithm FORM with sensitivity analysis in the space of original variables was proposed by Ji and Liao (2014).

Erzin and Cetin (2014) assessed 5000 homogeneous finite slopes with different slopes using artificial neural network (AAN) and multiple regression (MR) models. They found that AAN produces more reliable results than MR models.

In spite of all these efforts, there are still some modelling errors, like the ones due to lab and field work uncertainties, especially on parameters like the matric suction, which currently do not have systematic calibration procedures. Other errors are induced, for example, on the statistical treatment of rainfall intensity: the consideration of maximum daily or hourly values from the records which have a discrete handling, instead of the more realistic model through a continuous variable. There is an issue with the rainfall duration and the correlation between intensity and duration, which is not constant although sometimes is considered as a fixed number. The assumptions about the value of the target failure probability is other example of our imperfect knowledge to set an upper tolerable bound on the slope failure probability.

In the present study, the time-variant slope failure probability is calculated to identify the priority order to provide attention according to its relative value respect to the target limit and also considering the failure consequences. The target failure probability is obtained as a function of the expected loss due to failure consequences; this is especially important to discriminate cases with different failure consequences.

A simplified approach is proposed here to assess the priority level required for slopes for the attention for scarce funds allocation. Random soil properties and random rainstorm characteristics are analysed, in addition to the time dependent sampling of the rainfall conditions. In case the slope failure probability exceeds the target value, a mitigation action is assessed to reduce the slope failure probability.

2. Proposed formulation

The proposed formulation includes the soil stability analysis including uncertainties on some soil properties and rainfall characteristics. Cost data, for all the considered failure consequences, are also included. Later on, the rainfall precipitation model (bivariate intensity-duration distribution as derived from the available records for each site) are introduced and the soil-water characteristics are considered in the form of SWCC (soil-water characteristic curves) for the soil types above mentioned. Five random variables are considered: soil friction angle, the soil cohesion, the soil volumetric weight, the rainfall intensity and the rainfall duration. Later on, a

Monte Carlo simulation process is performed by considering a rainfall intensity per day, where each trial includes the variation of rainfall intensity sequence for several days. The slope safety factor is calculated by following the conventional slices stability analysis procedure. These safety factor is conditional to the randomly generated values of the variables and represents the slope safety for the considered trial and the specified day; there will be as many safety factors as trials are being performed for that day. Once the failure probability is calculated through the event where the safety factor is lower than 1 for that day, the process is repeated for all the days into the rainfall season.

The target failure probability is calculated by minimizing the present value of the expected life-cycle cost and it is compared to the previously calculated slope failure probability.

The number of trials for the MCS is the minimum number to have a stable value of the mean failure probability. The exercise is illustrated for 3 sites in Mexico with strong rainfall regime and, if the slope failure probability exceeds the target value, a mitigation work like the installation of anchors (Montoya, 2009) is proposed and modelled and the failure probability is re-calculated. Several arrays of anchors sizes and distributions are essayed, until the slope failure probability is lower than the target value. The formulation may be adapted to devise optimal retrofit strategies and to derive cost/benefit basis to quantify the trade-off between the cost of the mitigation works and the cost reduction on the expected failure costs.

2.1 Numerical simulation procedure

By considering the friction angle ϕ, the cohesion c, the volumetric weight γ, the rainfall intensity i and the rainfall duration d as random variables, the slope failure probability may be defined as:

$$P_f = \iiint_{\emptyset,c,\gamma} \iint_{i,d} (SF < 1 | \emptyset, c, \gamma, i, d) f(\emptyset, c, \gamma, i, d) \mathrm{d}\emptyset \mathrm{d}c \mathrm{d}\gamma \mathrm{d}i \mathrm{d}d \qquad (1)$$

Where SF is the safety factor, calculated for each trial. Given that the joint probability density function f is hard to be analytically determined, a simplified numerical procedure needs to be implemented. Given that the soil mechanical properties change according to the rainfall features and the rainfall intensity is related to the rainfall duration, the trials for the MC simulation may be structured as follows: first, simulate the rainfall duration, then calculate the rainfall intensity including the correlation with the rainfall duration. Later on, the soil properties are calculated as a function of the rainfall features and the random variation on these properties. In the last step of the trial, the slope stability is analysed and the failure probability is calculated.

The safety factor *SF*, at each trial, is calculated from the classical slices method (Fellenious, 1927), which resorts on the values of ϕ, c, γ, i and d, and for the geometry of the considered slope. The safety factor is based on the forces provoking and resisting the sliding throughout the considered circular failure path. The moment´s equilibrium may be expressed as:

$$SF = \frac{\sum[c'bR+(N)Rtg\varphi']}{\sum Wx-\sum S_m R-\sum Nf+\sum kWe\pm\sum Aa} \quad (2)$$

Where: c´ = effective cohesion force, b = slice base length, R= radius of the circular sliding surface, N = normal force to the sliding surface, ϕ´= effective friction angle, W= slice weight, x = horizontal distance from slice centroid to the center of rotation (centroid of circular failure path), S_m= friction force, A = resultant force from water, a = distance from the center of rotation to the water force.

A simplified flowchart describes the calculations, see Figure 1.

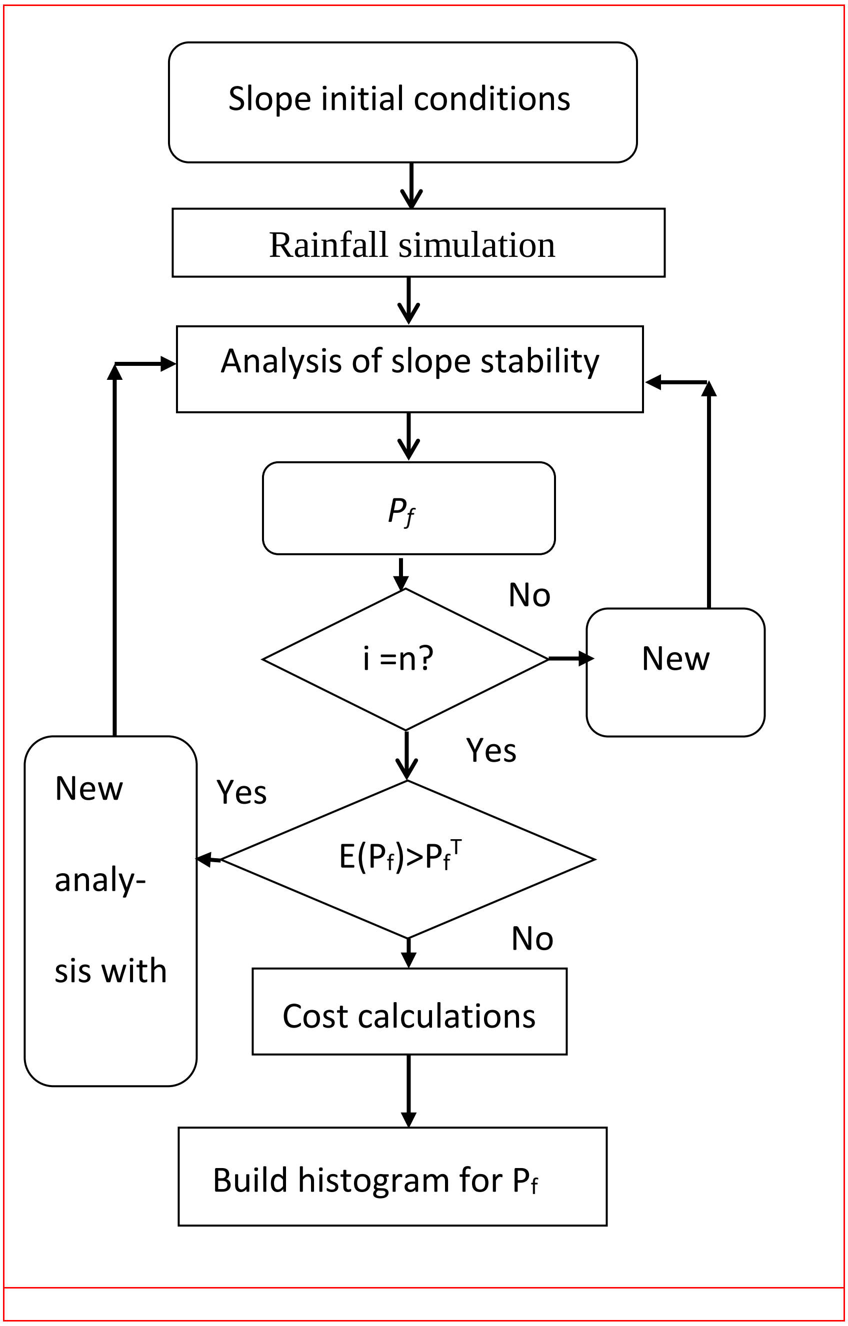

Fig. 1 Simplified flowchart of proposed formulation

The expected life-cycle cost $E(C_L)$ is calculated, according to:

$$E(C_L) = C_i + E(C_f) \qquad (3)$$

Where C_i is the slope initial cost and $E(C_f)$ is the expected present value of the cost of the failure consequences.

$$C_i = C_1 - C_2(lnP_f) \qquad (4)$$

Where C_1 and C_2 are constants that depend on the slope; specifically, C_2 is the cost to reduce the slope failure probability in one cycle of *ln*. The cost of failure consequences, C_f, includes the potential fatalities, property loss and economic consequences, for instances the user costs of highway detours if there is a highway near the slope.

The expected present value of the cost of failure $E(C_f)$ is (Ang and De León, 2005):

$$E(C_f) = PVF\ (P_f)C_f \qquad (5)$$

Where *PVF* is the present value factor which is expressed in terms of the annual discount interest rate *r* and the slope lifetime *T*:

$$PVF = [1 - exp(-rT)]/r \qquad (6)$$

The slope failure probability included in the expected life-cycle cost, and defined in Eq. (1), depends on the safety factor, *SF*, which is calculated by the slices method.

The target failure probability P_f^T is used, as a tolerable limit, to decide if the slope is stable ($P_f^T > P_f$) or requires mitigation works ($P_f^T > P_f$) to reduce the slope failure probability to keep it below its target value. The target failure probability is obtained from the minimization of the expected life-cycle cost.

$$\frac{\partial E(C_L)}{\partial P_f} = 0 \qquad (7)$$

$$P_f^T = \frac{C_2}{PVF(C_f)} \qquad (8)$$

Where *PVF* is the present value factor and C_f the cost of failure consequences.

2.2 Soil shear stresses and infiltration on slope soil

The shear stress on the soil, once a rainstorm starts modifying the soil structure, is one of the most important parameters on the soil stability analysis and it needs to be calculated in terms of the water pore pressure, effective cohesion and soil matric suction, among other parameters. These shear stress is obtained, according to Vanapalli et al., 1996:

$$\tau = c' + (\sigma_n - u_a)tg\varphi' + (u_a - u_w)\left(\frac{\theta - \theta_r}{\theta_s - \theta_r}\right)tg\varphi' \qquad (9)$$

Where:

τ = soil shear stress

c' = soil effective cohesion

σ_n = total normal stress

u_w = pore pressure (water)

u_a = pore pressure (air)

ϕ' = effective friction angle

θ_w = volumetric water content

θ_r = residual volumetric water content

$(\sigma_n - u_a)$ = net stress

$(u_a - u_w)$ = soil matric suction

The water infiltration into the soil, due to rainfalls, is calculated through the equation developed by Richards (1931):

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial H}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial H}{\partial y}\right) + Q = \frac{\partial \theta}{\partial t} \qquad (10)$$

where:

H = total pressure

k_x = hydraulic conductivity in the x- direction

k_y = hydraulic conductivity in the y-direction

Q = flow applied in the contour

θ = volumetric water content

t = time

The calculations are performed through the commercial software Geostudio (2019).

3. Illustrations

A typical slope in Mexico and 3 sites, with strong rainfall hazard in Mexico, was selected to show how the slope vulnerability changes under rainfalls with combined intensity-duration that results on a sudden increment on vulnerability. The slope is assumed to be the same and the 3 sites were selected according to the location of available meteorological stations at regions were usually occur strong rainfalls.

3.1 Description of the proposed slope

The slope is located at Zinacantepec, Mexico and the geometrical model, with the considered failure surface, is shown in Figure 2. The failure surface is identified after a series of preliminary analyses, with several assumed failure paths, under the condition of the minimum value of the safety factor.

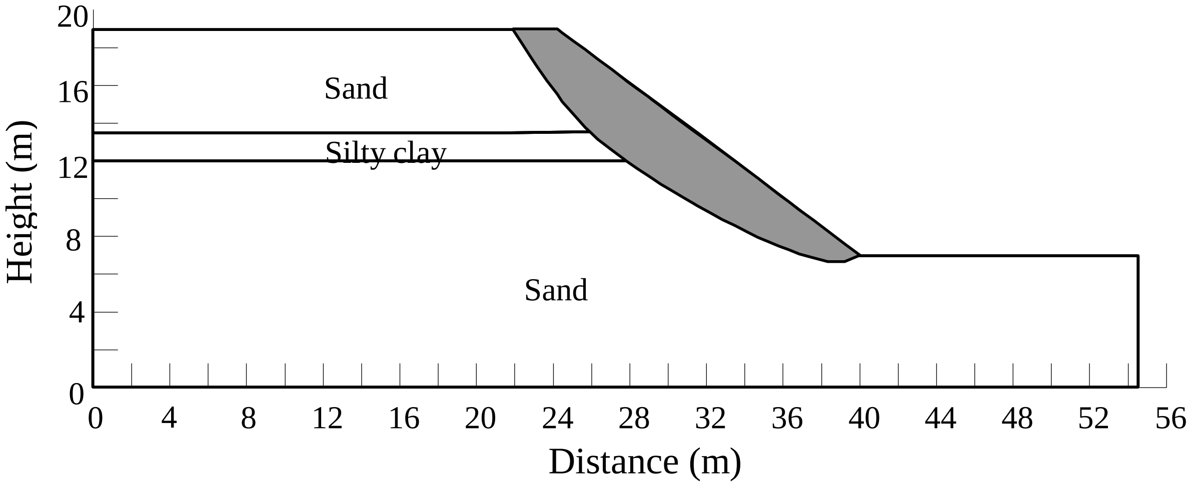

Fig. 2 Geometric profile of slope and failure surface

The statistics (mean and standard deviation) of the soil properties in its initial condition, are shown in Table 1. The initial conditions are dry conditions, before the rainstorms start.

Table 1 Statistics of soil properties, initial condition

Soil	Statistics	Cohesion	Friction angle	Volumetric weight
Sand	Mean	8 kPa	35°	18 kPa/m^3
	Standard deviation	1.6 kPa	3°	0.15 kPa/m^3
Silty clay	Mean	10 kPa	25°	19 kPa/m^3
	Standard deviation	2 kPa	2°	0.16 kPa/m^3

3.2 Description of meteorological hazards for the illustration

Three states in Mexico have the strongest rainfalls ever recorded: Chiapas, Tabasco and Morelos. The above described slope is considered to be located at sites within these 3 states, for illustration purposes.

For example, the recorded rainfalls in Chiapas show that, from 1962 to 2017, there were 14 days with more than 100 mm/h rainfall and 3 other times the rain duration longed 13 consecutive days (De León and Pérez, 2016).

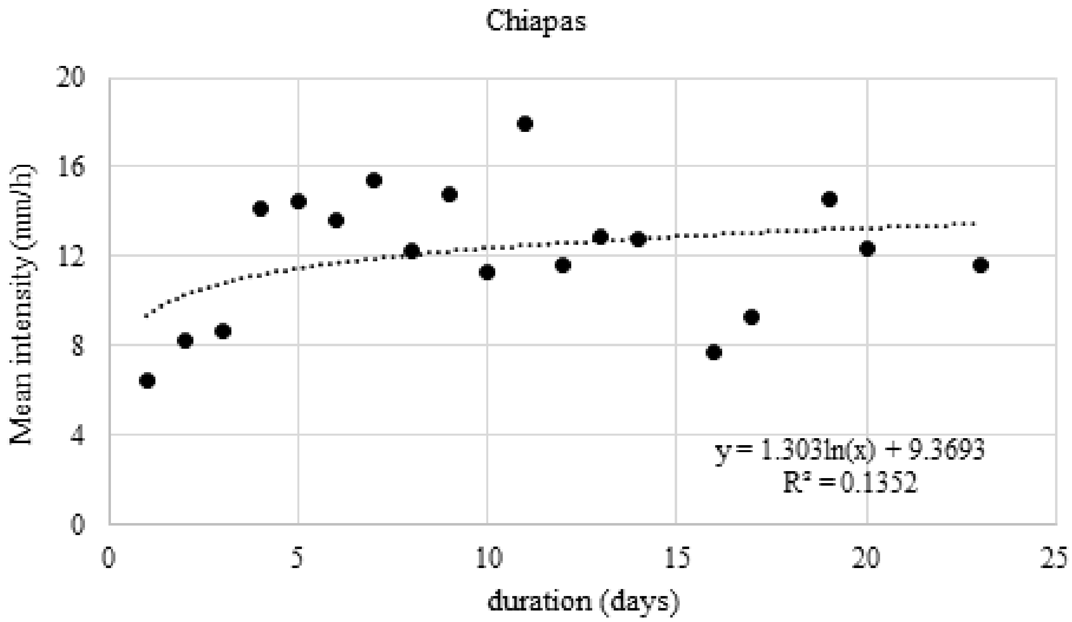

Fig. 3 Mean intensity for several duration for Chiapas slope

Goodness of fit tests were performed for these records and it was found that the best fit is exponential for the rainfalls intensity in Chiapas. Figure 4 shows the actual and expected frequencies for Chiapas rainfall intensities.

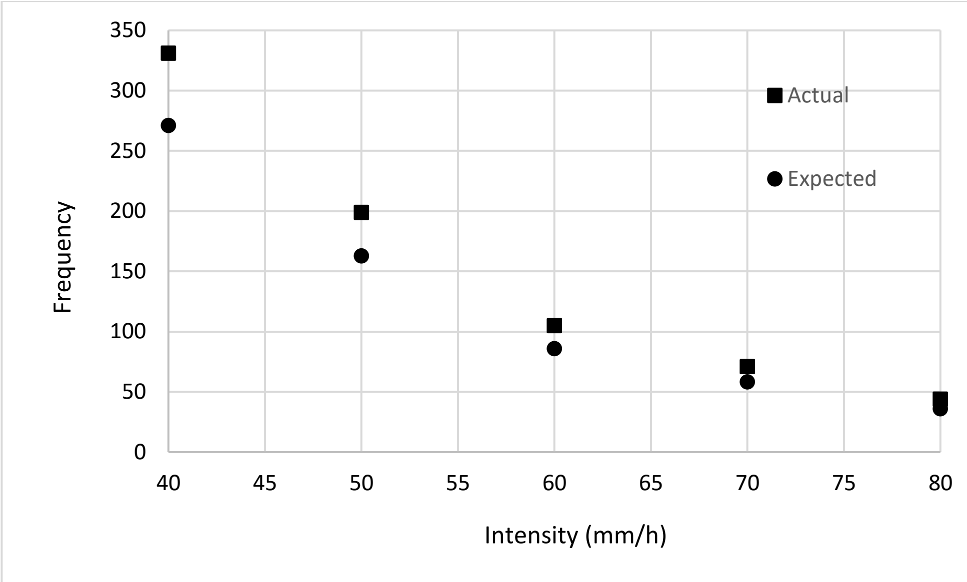

Fig. 4 Actual and expected exponential frequencies for Chiapas rainfall intensities

Given the correlation between intensity and duration, the following relationships between mean intensity conditional to the duration, Equation (11), and standard deviation of intensity for a given duration, Equation (12), were obtained for the MCS calculations:

$$E\big((i|d)\big) = 1.303\ln(d) + 9.369 \qquad (11)$$

$$\sigma_i = 2.764\ln(d) + 4.865 \qquad (12)$$

Once the MCS process was performed, as above described, it was observed that, without rainfalls, the slope safety factor is 4.76 (see Figure 5). This was the slope initial condition.

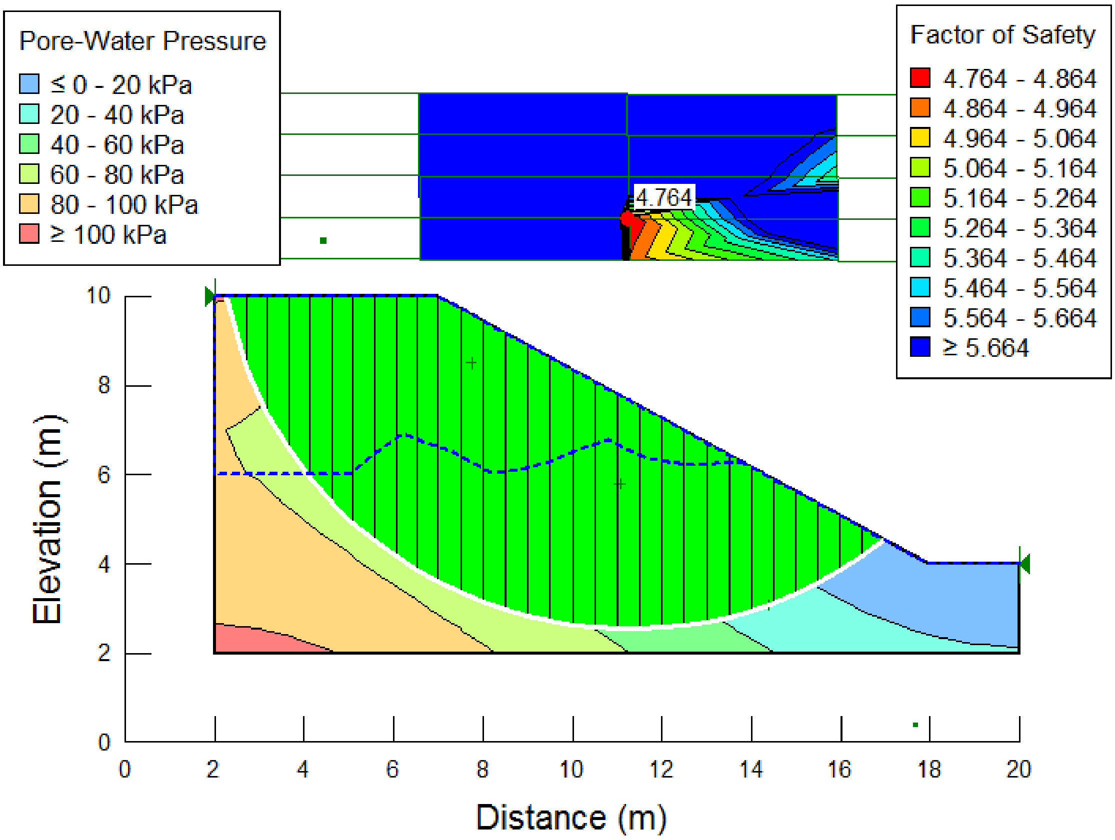

Fig. 5 Screen view of Chiapas slope for a day with no rainfall conditions

However, during the intense rainfall days, the safety factor was reduced to values lower than 1, see Table 2, and the failure probability increased to a very high value as 0.71.

Table 2 Safety factor and failure probability for slope at Chiapas, for intense rainfall days

Day	Rainfall Intensity (mm/hr)	Safety factor	Failure probability
15	44	0.38	0.71
16	44	0.38	0.71
17	11	1.39	0.35
18	11	1.39	0.35
19	4	2.55	0.07
20	6	2.06	0.16

21	6	2.06	0.16
22	6	2.06	0.16
23	6	2.06	0.16
24	6	2.06	0.16
25	10	1.49	0.32
26	24	0.66	0.62
27	24	0.66	0.62

The safety factor was plotted against time, for a trial of the MCS process, as shown in Figure 6.

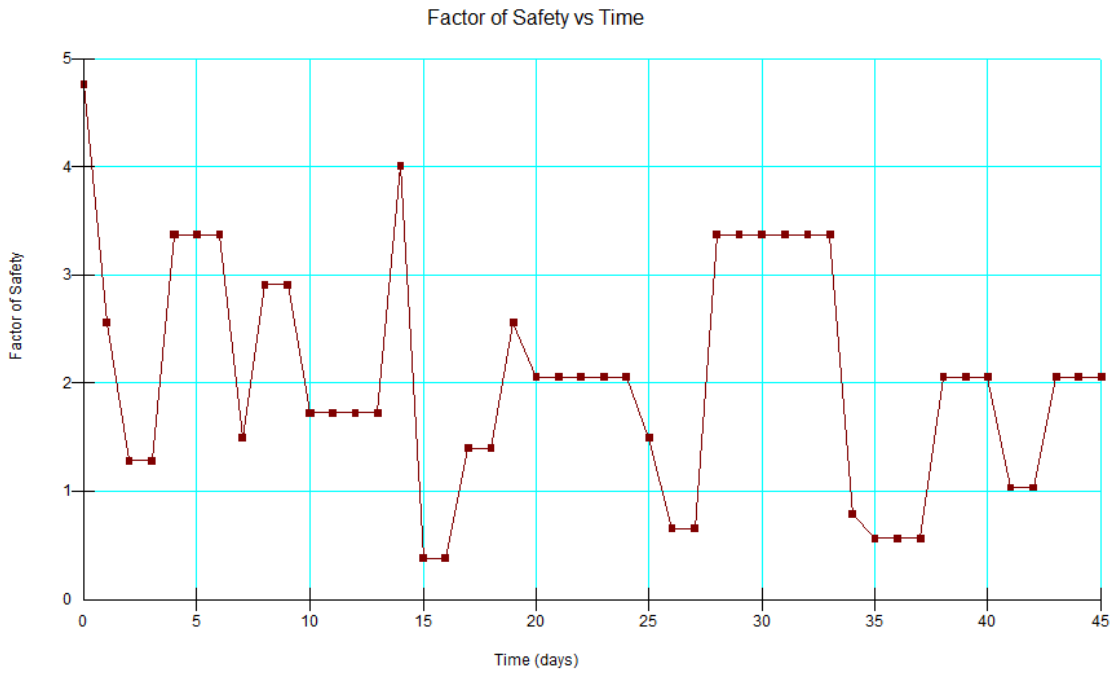

Fig. 6 Screen view of safety factor for Chiapas slope for a trial of 45 days

The slope annual failure probability is 0.71, which is compared to the target value, from Equation (8). The target failure probability is calculated by considering a failure cost C_f = 1.3 million USD and C_2 = 0.6 million USD; with these figures, the target value is 0.037. It is observed that the slope failure probability (0.71) is too high for the allowable (target) value (0.037). Therefore, the slope requires mitigation actions.

As previously performed for Chiapas, the relationships between mean intensity vs: duration and standard deviation of intensity vs: duration, are calculated for the Tabasco slope. Tabasco has rainfall intensity records, between 1943 and 2018, with more than 100 mm/hr in 20 non consecutive days and, in 5 of these days, the registered intensity was 185.5 mm/hr; see Figure 7.

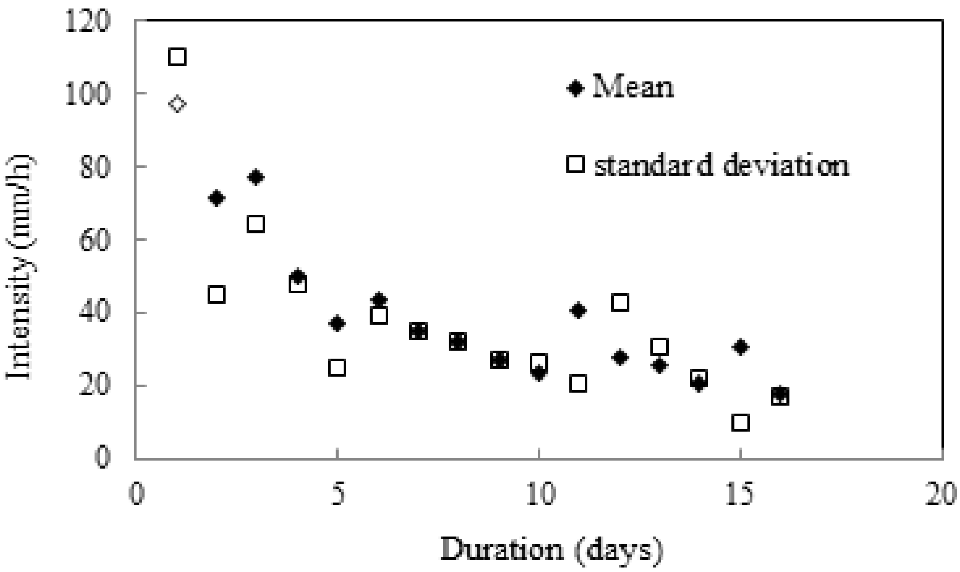

Fig. 7 Mean and standard deviation of intensities vs: duration for Tabasco

$$E(i|d) = -26.78\ln(d) + 92.539 \quad (13)$$

$$\sigma_i = -25.82\ln(d) + 86.583 \quad (14)$$

The MCS was performed for the slope located at Tabasco and, in the 10^{th}. day of rainfalls, the safety factor was 0.29 and the corresponding slope annual failure probability is 0.78, again too high for the target value obtained for reasonable failure costs. Figure 8 shows a screen shot of a sample of the water pore pressure and the safety factor space distributions.

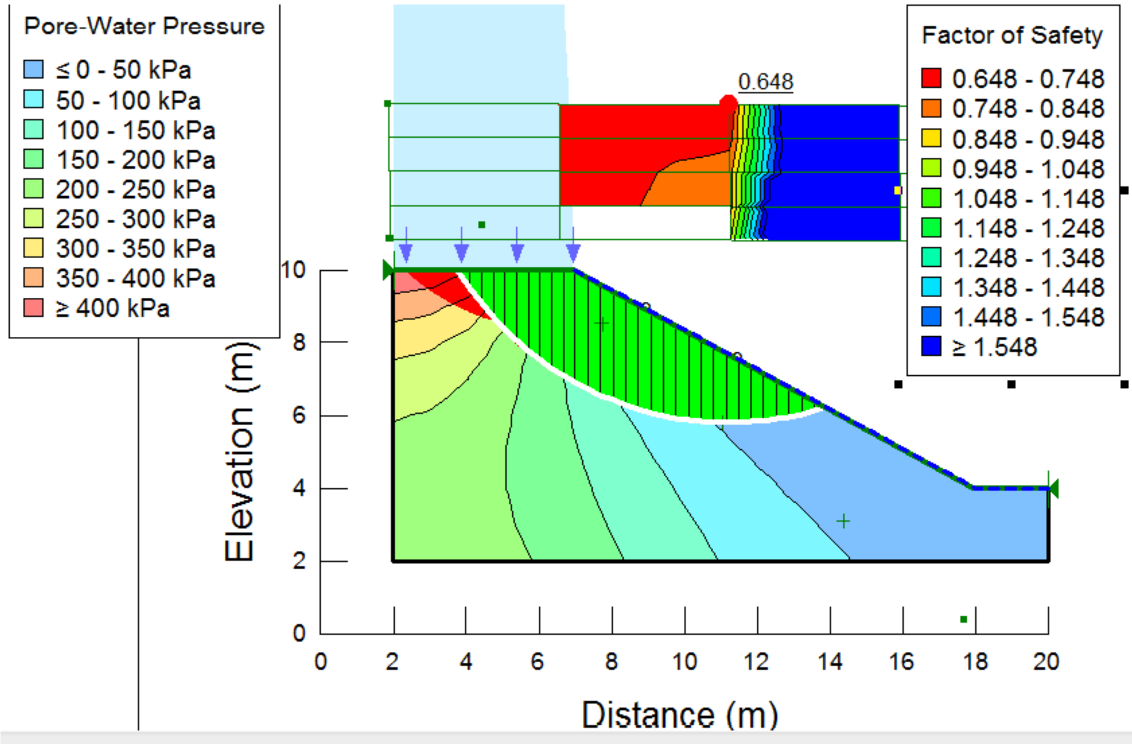

Fig. 8 Screen view of a trial for 1st. day with rainfall for Tabasco slope

Finally, the same relationships were calculated for Morelos. In Morelos, within the recording period from 1924 to 2014, there was a rainfall series of 12 days with an intensity of 186.5 mm/hr. The mean and standard deviation of the intensity vs: the duration is shown in Figure 9.

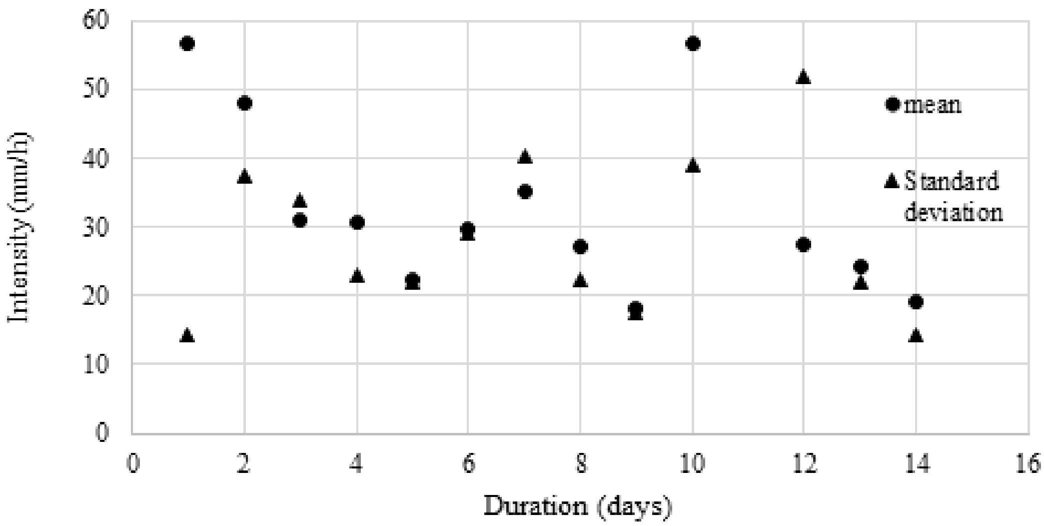

Fig. 9 Mean and standard deviation of intensities vs: duration for Morelos

$$E(i|d) = -9.755 \ln(d) + 49.874 \quad (15)$$

$$\sigma_i = 2.054 \ln(d) + 24.633 \quad (16)$$

After performing the calculations for the evolution of the safety factor, the series of safety factors for a 50 days period, is shown in Figure 10.

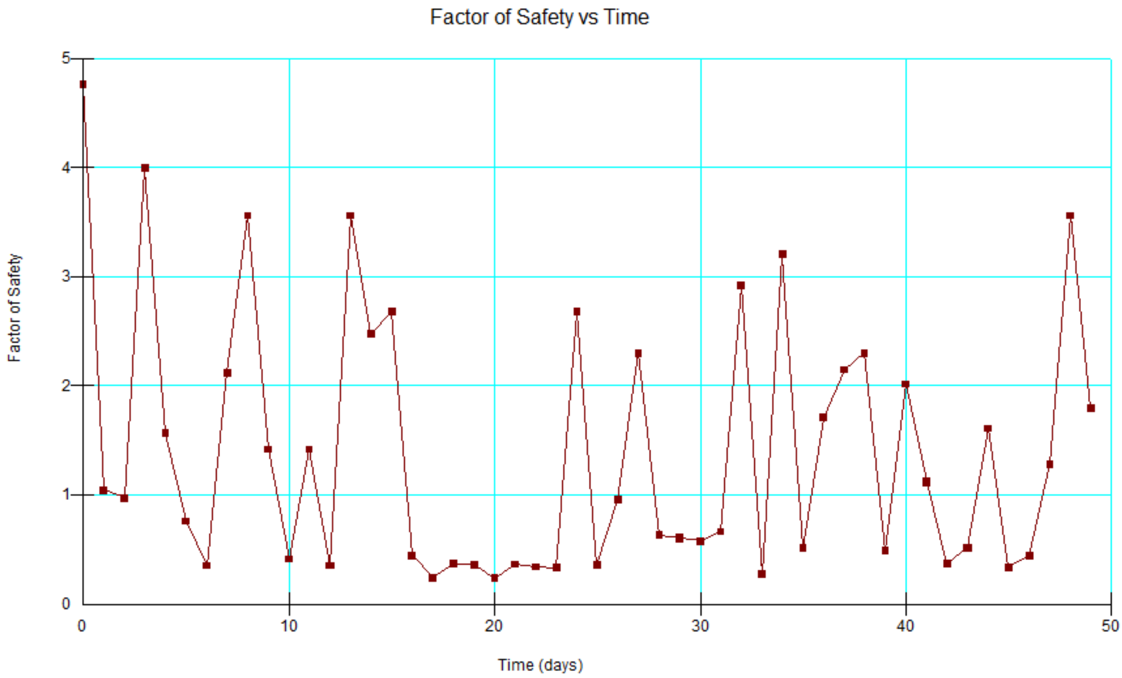

Fig. 10 Screen view of safety factor, for a 50-days period, for Morelos slope

The minimum value of the slope annual failure probability is 0.74 and the corresponding safety factor is 0.23. Figure 11 shows the time variation of the failure probability, for a 20 days period, for Tabasco and Morelos.

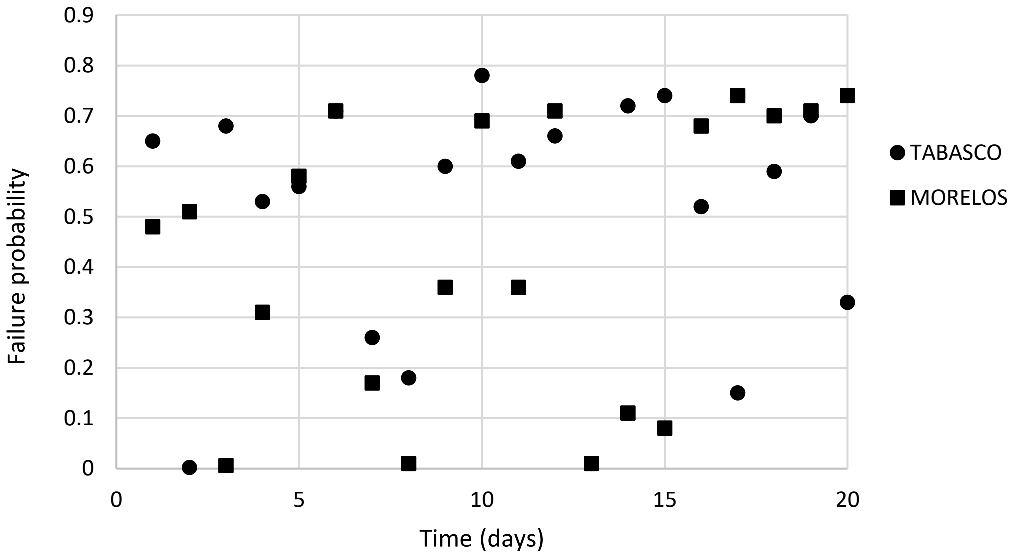

Fig. 11 Failure probability, for a 20-days period, for Tabasco and Morelos slope

As observed, these slopes require mitigation measures in order to increase the safety factor and, consequently, reduce the failure probability.

4. Mitigation actions

4.1 Alternatives

One of the mitigation works may be the slope cut (reduction on slope inclination angle), the covering of concrete on top and or the sides of the slope, the installation of steel anchors to reinforce the soil, the colocation of drainage pipes and the installation of a steel grid on the slope surface, among others. The selected alternative has to be modelled to recalculate the new failure probability, and also the corresponding costs of the works, to estimate the cost/benefit and compare the alternatives, looking for the optimal mitigation work. In this paper the option of anchors installation is selected to be applied to the Chiapas slope.

4.2 Anchorage applied to Chiapas slope

The Eq. (2), and therefore the method of slices, is slightly modified to include the anchor forces *AN* to resist the sliding trend on the failure surface (Instituto Vasco de Seguridad y salud

Laborales. 2004; Monroy, 2007). Several anchors arrays may be essayed in order to make an efficient use of these forces to increase the safety factor. The main requirement is that all anchor bars should cross the slope failure path at several locations. See figure 12.

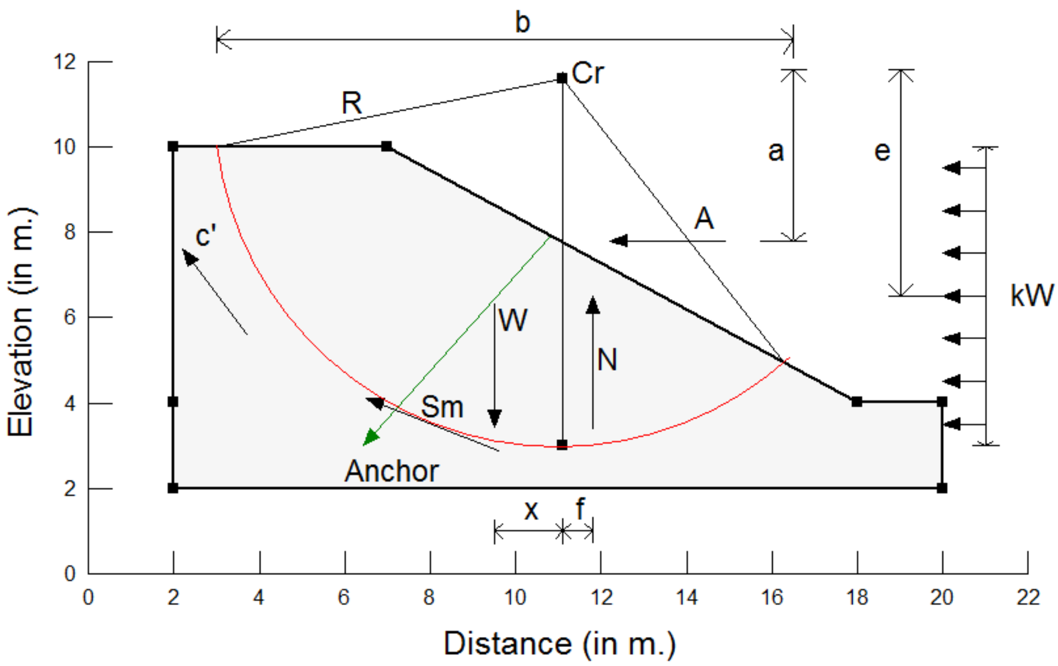

Fig. 12 Slope analysis with anchor bars

The new safety factor, SF_A, including the anchor forces, is (Valladares, 2015):

$$SF_A = \frac{\sum[c'bR+RN(tg\varphi')]+AN}{\sum Wx-\sum S_m R-\sum Nf+\sum kWe\pm\sum Aa} \quad (17)$$

As an example, it will be illustrated the mitigation applied to the Chiapas slope:

The installation of a series of 8 anchorage bars, 5/8”ϕ, is proposed, as illustrated in Figure 13. The minimum safety factor increases to 2.23, a substantial increment respect to the minimum of 0.78, without anchor bars. The corresponding failure probability becomes 0.0372 which meets the target value.

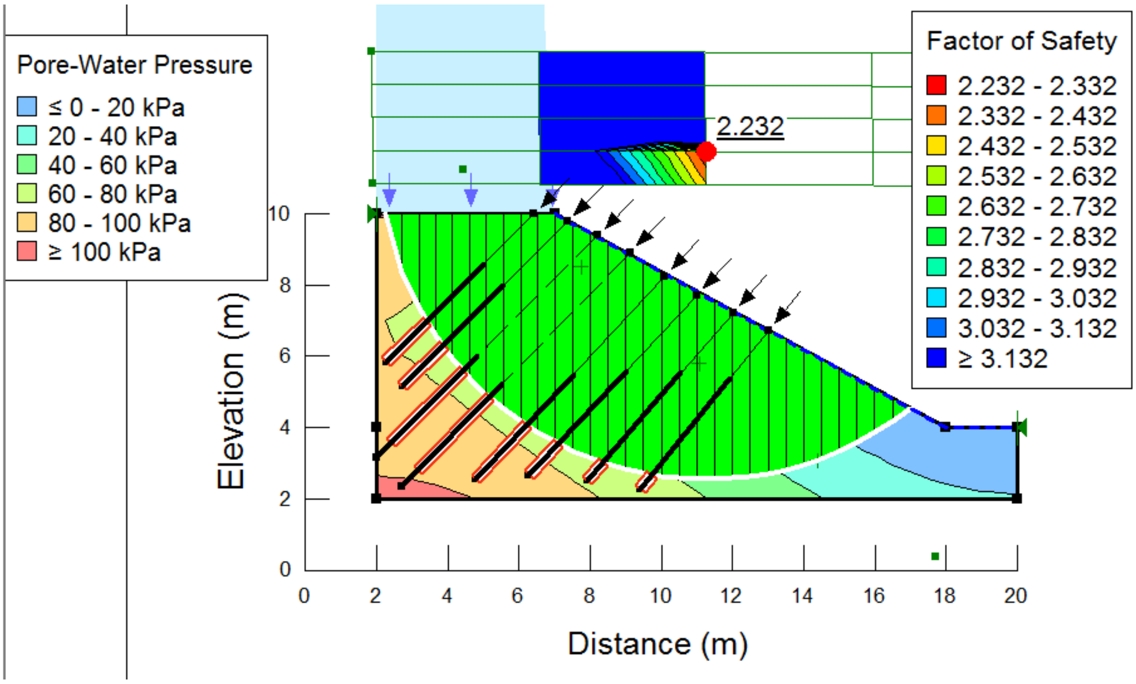

Fig. 13 Slope analysis with anchorage bars for Chiapas

The cost of material and installation of the anchor bars about 0.2 million USD. By following the same procedure, other mitigation methods may be essayed and their corresponding costs may be calculated in order to select the optimal mitigation work, i. e., the one that best balances the costs with the reduction on failure probability.

5. Conclusions

A simulation procedure to predict the time-dependent failure probability of slopes under strong rainstorms, and to assess the economical effectiveness of mitigation measures in case they are needed, has been proposed and illustrated for 3 sites in Mexico. The procedure includes the uncertainties on the soil resistance properties and on rainfall intensity and duration and it combines the safety and cost assessments.

For the slope and rainfall environment considered here, the safety factor may be as low as 0.38, and the corresponding annual failure probability as high as 0.78, like the results obtained for Tabasco.

Critical slopes (with vulnerable geometries and soft soil types) and strong rainfalls (with the combination of moderate/high intensities and long duration) increase the slope failure probability and may require the implementation of mitigation measures, like the one applied to the Chiapas slope.

For slopes similar to the one illustrated here, located at Chiapas, Tabasco and Morelos mitigation works are mandatory and the proposed procedure may help to identify the optimal work type for each condition.

The use of anchor bars mitigates the slope vulnerability, as illustrated for the Chiapas slope, and other mitigation works should be assessed to develop optimal recommendations. For the Chiapas slope, the use of 8 anchorage bars, with 5/8" diameter, with a cost of 0.2 million USD, reduces the failure probability from 0.78 to 0.037, which is enough to meet the target failure probability.

The procedure may be extended to generate prioritization on the attention to slopes with different geometries, soils composition and rainfall patterns around the country.

It is recommended to extend this work to derive optimal recommendations for mitigation works, to generate landslides hazard/risk maps, for each region, and to propose risk-based guidelines to protect critical infrastructure or populations located on landslide-prone neighbourhoods.

Acknowledgments

The research described in this paper was financially supported by the CONACYT (Consejo Nacional de Ciencia y Tecnología) under the grant "Problemas Nacionales" 247783 from Mexico. The authors also thank Cenapred (Centro Nacional de Prevención de Desastres) for the information provided and the organization of dissemination meetings.

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